Nondispersing Trojan-Like Wavepackets on Langmuir Type-(2) Click-Clack Balls Oscillatory Model Trajectories in Helium Atom and Quantum Dots

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Some time ago we have discovered that placing the Langmuir trajectories [1] of the type one i.e those in the "Hoop Earrings" configuration in a combination of the symmetry augmented Circularly Polarized (C.P.) electromagnetic field and the magnetic field perpendicular to the planes of the both electron parallel circular motions results in classical stabilization of the resulting Langmuir trajectories which therefore can support the stable non-dispersing quantum Trojan Wave Packets [2].

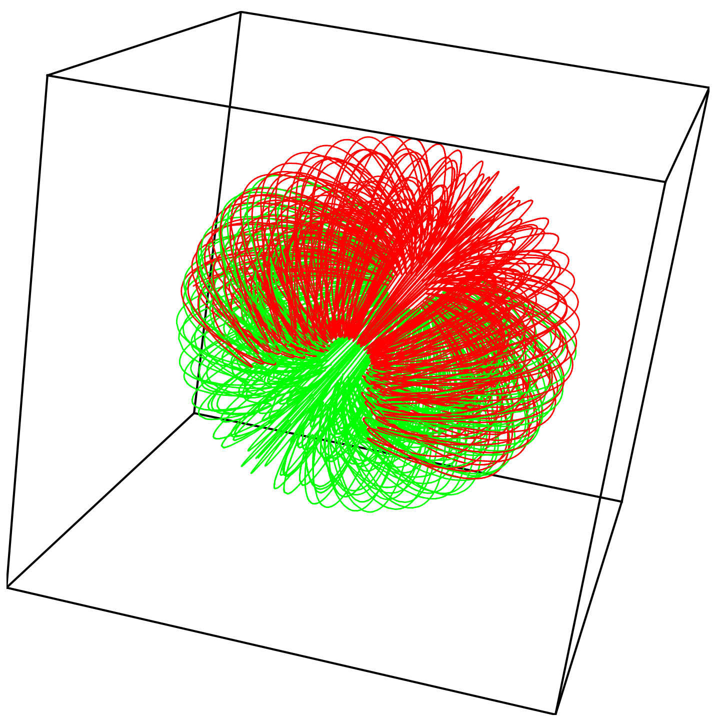

Fig. 1. Perturbed Langmuir two-electron (two-color) trajectories executing the periodic motion of the Click-Clack Balls with the slow click-clacking plane rotation resulted from the intuitive initial condition of the centrifugal forces and the Coulomb forces balance for the Helium nucleus of the charge $Z = 2$ i.e. $m\omega^2 r = Z/r^2 - 1/(2r)^2$ leading to otherwise circular trajectories if the initial electron velocities were in the opposite direction. Like for the Click-Clack Balls collision the electrons are approaching each other and reverse the velocities due to the wall effect of the Coulomb repulsion. The initial electron positions for the unit ω are $[\pm(7/4)^{1/3}, 0, 0)]$ and the augmented velocities of both $[0, (7/4)^{1/3}, \pm 0.001]$. Small augmented z-components of the initial velocities are added to obtain the slow bouncing plane rotation. Near perfect original Langmuir trajectories (sharp lines of bouncing on semi-circular trajectories) and without any motion plane rotation can be obtained by scaling the initial velocities only by a factor of 1.09 and cooling the z velocities to 0. The similar trajectories may be obtained by adding the resonant to the field-free motion Linearly Polarized (L.P.) field and the magnetic field which is not changing the trajectory geometry while properly adjusted. In the case of the magnetic field addition the plane of the L.P. field must rotate with the half of the cyclotron frequency (the "L.P." field is therefore a superposition of two counter-rotating C.P. fields of a different frequency) to balance the Lorentz force asymmetry and the packets exist in the L.P. plane rotating frame.

We have also considered several other configurations of electrons and fields leading to the existence of the stable, shape invariant Trojan or Trojan-like wave packets of one or two-electrons moving on circular or near-circular orbits. The common feature of those configurations is that the large angular momentum of the electrons is precisely adjusted to the Coulomb, Lorentz and the electromagnetic field driving force so each of the electron orbit stays circular and therefore frozen in the rotating frame.

Here we show that the Langmuir trajectories of the type two i.e. those corresponding to the popular toy, the Click-Clack Balls when two electrons are moving in one plane on the semi-circular trajectories with the opposite angular velocity, they bounce from each other, reverse the velocities and continue bouncing again and again also support such packets. While the field-free two-electron trajectories are semi-circular for each electron the properly tuned resonant Linearly Polarized (L.P.) electromagnetic field is acting on each electron on a semi-circle like it was a Trojan electron in the Hydrogen atom. While the L.P. field is the superposition of two counter-rotating C.P. fields each component of the field alternately couples to the electron with the proper sign of the angular velocity. To stabilize and confine the system further the static magnetic field can be added in addition to the resonant L.P. field. In that case to preserve the symmetry of the trajectories the polarization plane of the L.P. must rotate with the half of the cyclotron frequency corresponding to the magnetic field to balance otherwise asymmetric Lorentz forces acting differently of each of two electrons. Therefore in the former case the trajectories and the corresponding wave packets moving along them exist in the frame rotating with the plane of the L.P. field polarization i.e. the electron bouncing point is rotating in the laboratory frame.

Unlike for the Trojan wave packets the packets are not perfectly shape-invariant due to the reoccurring quantum collisions but still are highly confined and non-dispersing.

We use the generalized Gaussian ansatz

$$\psi = N \exp[-\sum \mathcal{M}_{ij} \tilde{x}_i \tilde{x}_j], \tag{1}$$

$\tilde{x}_i = x_i - x_{0i}(t)$ for the packet wave function and solve the equations for the localization matrix $\boldsymbol{\mathcal{M}}(t)$ together with the classical equations of the motion.

We find the nondispersing wavepackets in the joined combination of the external L.P. field and the static magnetic field and the frequency tuned to the natural frequency of the closed periodic orbits drawing the single electron semi-circles. Numerical simulations using the split operator method for the 3D Hartree approximation as well as our recently developed Time Dependent Quantum Diffusion Monte Carlo Method are also provided.

References

1. I. Langmuir, “The Structure of the Helium atoms”, Phys. Rev. **17**, 339 (1921).
2. M. Kalinski, L.Hansen, and D. Farrelly, “Nondispersive Two-Electron Wave Packets in a Helium Atom”, Phys. Rev. Lett. **95**, 103001-103004 (2005).