

# Nonlocal Limit of Cherenkov Radiation in Hyperbolic Metamaterials

Hao Hu,<sup>1†</sup> Xiao Lin,<sup>2†</sup> Dongjue Liu,<sup>1</sup> Patrice Genevet,<sup>3</sup> Baile Zhang,<sup>2,4</sup> and Yu Luo<sup>1\*</sup>

<sup>1</sup>School of Electrical and Electronic Engineering, Nanyang Technological University, Nanyang Avenue, Singapore 639798, Singapore

<sup>2</sup>Division of Physics and Applied Physics, School of Physical and Mathematical Sciences, Nanyang Technological University, Singapore 637371, Singapore

<sup>3</sup>Universite Cote d'Azur, CNRS, CRHEA, rue Bernard Gregory, Sophia Antipolis 06560 Valbonne, France

<sup>4</sup>Centre for Disruptive Photonic Technologies, Nanyang Technological University, Singapore 637371, Singapore

<sup>†</sup>These authors contributed equally to this work

\*Corresponding author email: LUOYU@ntu.edu.sg

## Abstract

A charged particle travelling faster than the phase velocity of light in a substance can emit Cherenkov radiation (CR). This phenomenon was experimentally discovered by Cherenkov in 1934 [1] and theoretically explained by Frank and Tamm's work in 1937 [2]. However, CR in conventional isotropic crystals requires the particle velocity beyond a threshold so called Cherenkov threshold (i.e.  $c/n$ ). Due to finite refractive index  $n$  of the nature material, the Cherenkov threshold is high up to hundreds of keV, limiting the applications of Cherenkov devices (e.g. Cherenkov detectors or Cherenkov emitters).

A recent work revealed that CR in the hyperbolic metamaterials (HMMs) may offer the possibility to 'remove' the Cherenkov threshold [3], suggested by effective medium theory (EMT). While this theory is ubiquitous in the deep subwavelength range, it leads us to specious conclusions in some special situations. Especially, when optical modes involve much larger wavevectors compared with those in bulk media, metamaterials exhibit strong spatial dispersion. This regime, also called non-local regime, gives a more accurate description of the material dielectric response. How the nonlocal effects modify behaviors of CR such as the Cherenkov threshold still remains elusive.

In this work, we explored the nonlocal effects on the CR in the HMM, shown as Fig. 1(a). We prove the existence of Cherenkov threshold in realistic multilayer structure, as suggested in Fig. 1(b) and (c). On the one hand, the finite thicknesses introduces a cut-off on the maximum achievable wavevector  $k_y^{cf}$  (seeing Fig. 1(d)-(f)), inducing non-zero Cherenkov threshold. On the other hand, as geometric features approach to the Thomas-Fermi screening length  $\lambda_F$ , the charge screening effect inside the metal sets an ultimate bound on the minimum Cherenkov threshold as  $v_{th} = \beta = \sqrt{\frac{3}{5}} v_F$ , where  $v_F$  is Fermi-velocity (seeing Fig. 1(g)).

## Images

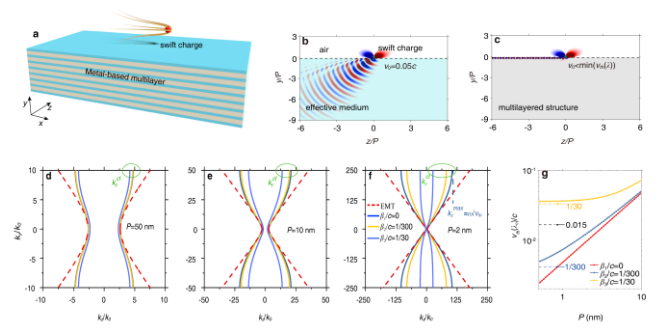


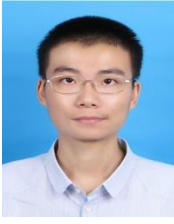
Fig. 1. (a) Structural schematic. A swift electron moves with a velocity of  $\bar{v} = \hat{z}v_0$  and its trajectory parallel to the interfaces of a multilayer periodic structure. The unit cell of the periodic structure is composed by an Ag slab and a SiO<sub>2</sub> slab, with the thickness of  $d_1$  and  $d_2$  respectively; the pitch of unit cell is  $P = d_1 + d_2$  ( $P \leq 50$  nm), which is much smaller than the interested wavelength  $\lambda$  in free space ( $\lambda \in [0.1 \text{ } 10] \mu\text{m}$ ). For all cases, we set  $d_1/P = 0.4$ . (b-c) Field distribution of Cherenkov radiation in the time domain. The realistic multilayer structure in (a) is adopted for (c), while it is replaced by an effective homogeneous hyperbolic material for (b). Here  $P = 25$  nm, and  $v_0 = 0.05c$ . (d-f) is the dispersion relation of the hyperbolic metamaterial in effective medium approximation, and realistic structure with different level of nonlocal charge screening effect. Here the period is varied from 50 nm to 2 nm. (g) The determined Cherenkov threshold at the wavelength  $\lambda_0 = 1 \mu\text{m}$ , in realistic structure with different level of nonlocal charge screening effect.

## Acknowledgement

This work was supported by Singapore Ministry of Education Academic Research Fund TIER 2 under Grant No. MOE2015-T2-1-145.

## Recent Publications

Hu, H.; Zhang, J. L.; Maier, S. A.; Luo, Y., Enhancing Third-Harmonic Generation with Spatial Nonlocality. *ACS Photonics* 2018, 5 (2), 592-598.



---

## Biography

Hu Hao received his Bachelor of Engineering in Zhejiang University, China, in 2016. He is currently a PhD student at the Photonics Centre of Excellence OPTIMUS, Nanyang Technological University, Singapore. Hao does research in Electrical and Electronic Engineering. His research interests are quantum and nonlocal plasmonics, Cherenkov radiation and metamaterials.

Email: [E160039@e.ntu.edu.sg](mailto:E160039@e.ntu.edu.sg)

---

## References:

- [1] P. Cerenkov, "Visible light from pure liquids under the impact of gamma-rays," *Cr Acad Sci Urss* **3**, 451-457 (1934).
- [2] I. Frank, and I. Tamm, "Coherent visible radiation of fast electrons passing through matter," *Cr Acad Sci Urss* **14**, 109-114 (1937).
- [3] F. Liu, L. Xiao, Y. Ye, M. X. Wang, K. Y. Cui, X. Feng, W. Zhang, and Y. D. Huang, "Integrated Cherenkov radiation emitter eliminating the electron velocity threshold," *Nat Photonics* **11**, 289-+ (2017).